OVERVIEW OF PROSPECTIVE MATHEMATICS TEACHERS' PROBABILISTIC THINKING

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This paper presents an overview of the models of probabilistic thinking constructed by 583 prospective mathematics teachers in Mendoza, Argentina. The goal was to gain insight into the personal meanings that these future teachers attribute to random phenomena and to the estimation of their probability. This knowledge should enable one to design the probabilistic education of prospective teachers from more appropriate conceptual and didactic standpoints, and thus contribute to the statistical literacy of secondary school pupils (Cardeñoso, Azcárate & Serradó, 2008; Meletiou-Mavrotheris et al., 2008; Carmichael, Callingham, Watson & Hay, 2009; Vega, Cardeñoso & Azcárate, 2010; 2011). For this reason all the prospective mathematics teachers responded to a twenty-four item questionnaire which was based on the category system proposed by Cardeñoso (2001). The responses were analysed using multivariate statistical techniques. These revealed four levels or trends in probabilistic thinking which fitted the descriptive labels of deterministic, personalistic, uncertainty, and contingent.

INTRODUCTION

In the teacher training centres of the province of Mendoza, Argentina, the mathematics teaching staff are currently immersed in a process of curricular transformation. It is hoped that the new designs will lead to substantial changes that can contribute to improving mathematics teaching and learning at the higher levels of education, including of course secondary education. This reform emerged as a necessity in response to the poor academic performance of pupils leaving secondary school, particularly in their competences in probability and statistics.

From our educational perspective, we understand teachers' Professional Knowledge to be a system of ideas that is in evolution from the simple to the complex. In this sense, the purpose of the present work was to discover what beliefs and conceptions the pupils have regarding randomness and probability. This would allow us to work towards developing new and different ways of thinking that might contribute to pupils' statistics education, and consequently to the cultural, scientific, and technical level of citizens.

THEORETICAL FRAMEWORK

From the perspective of the mental models proposed by Johnson-Laird (1983), people see the outside world through the mental representations that they build of it, which that author denominates simply "mental models".

The primary source of these mental models is perception. But they may also be constructed from discourse or be the result of a person's imagination. The relationship between mental models and discourse or perception is reciprocal: mental models are built from discourse or perception, and, in turn, the interpretation of that discourse depends on the mental models that one is capable of building. However, the construction of these models will be constrained by the structure of how the individual perceives or conceives of the world, by their pre-existing knowledge, and by their need to maintain their cognitive system free of contradictions.

People's mental models may be deficient in several aspects. They may include unnecessary, incorrect, or contradictory elements. But they must be functional. When a mental model has been useful for a person on several occasions, it is possible that all or some of its parts become stored in long-term memory, thereby adding to the person's baggage of knowledge. In this sense, Nersessian (1992) considers mental models to be intermediate levels of analysis between the phenomenon and the resulting final conceptual model. The theory based on mental models can help understand the phenomena of probabilistic reasoning since it was designed to explain the higher processes of cognition, especially comprehension and inference (Johnson-Laird, 1994; Johnson-Laird, Girotto, Legrenzi, Legrenzi & Caverni, 1999).

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When teaching with the use of textbooks and their different treatments of randomness (Azcárate, Cardeñoso & Serradó 2003), the teacher presents the pupils with certain conceptual models, and expects them to build mental models that will allow them to give scientifically accepted meanings to those conceptual models. In turn, the pupils' models must correspond appropriately to the natural phenomena or systems being modeled. From a cognitive point of view, the goal of this teaching is, through the professional management of the conceptual models, to get the pupil to build complex mental models that adequately match the systems or phenomena being studied.

The present research takes as referents previous work on teacher training in relation to randomness and probability, beginning with that of Azcárate & Cardeñoso (1994), and completed with that of Azcárate (1995) and Cardeñoso (2001). We propose to characterize the personal meanings that the pupils attribute to the field of indeterministic phenomena, referring to random events and probability estimation (Azcárate, Cardeñoso & Porlan, 1998; Azcárate & Cardeñoso, 2003). We start from the system of categories for the recognition of randomness and probability estimation proposed by Cardeñoso (2001). This was designed for and applied to in-service primary school teachers. The categories it uses are: *causality, multiplicity, uncertainty, subjectivity, contingency, Laplacian, frequency, equiprobability,* and *experiential*. We used this system of categories to prepare a questionnaire which we applied to 583 prospective mathematics teachers.

METHODS AND RESULTS

The questionnaire was answered by 583 prospective mathematics teachers in the province of Mendoza, Argentina. The students' responses were analysed by *k*-means clustering followed by a discriminant analysis. This confirmed the correct classification of 93.3% of the students. Four clusters emerged and, in accordance with their characteristics and in hierarchical order, were labelled: *deterministic, personalistic, uncertainty,* and *contingency*. These four clusters groups defined four levels of probabilistic thinking, similar to that described in prospective biology teachers (Moreno, Cardeñoso & González-García, 2012, 2013).

Deterministic

This is the group with the lowest level of randomness recognition, a fact which led to its name. When they deny randomness, they do so from a dual viewpoint of the categories of multiplicity, which considers the multiplicity of results of the phenomenon, and of uncertainty, which considers the unpredictability of the phenomenon. Both of these categories reached their maximum values in this group. This was a surprising result since one might think that students can discriminate random from non-random events or phenomena, but instead it was found that this group failed to distinguish which is which. In contrast, subjectivity, a category which takes beliefs and life experiences into consideration, is the one which is least used by this group. Regarding probability estimation, the strategy they followed has two facets. One is that of additive contingency, in which they compare favourable with unfavourable cases in the occurrence of the phenomenon. The other is that of equiprobability, a category whose values exceeded the overall mean value even though the majority of the situations posed involved sample spaces that were not equally probable. This thus appears to be more a bias than an appropriate argument relative to the situations put forward. Their use of the experiential category was similar to the mean, i.e., they justify the estimation of probability on the basis of their own experience.

Personalistic

This is the third group in the recognition of randomness. There stands out their use of subjectivity in the assertion of randomness. Indeed, this category reached its maximum value in this group. Although uncertainty had a greater presence, the value it reached was below the mean. Causality and multiplicity reached their minimum values in this group. When these respondents incorrectly reject randomness, they argue on the bases of causality and subjectivity, categories that reach maximum values in this group. Determinism in this group is therefore closely associated with causality and subjectivity. They estimate probabilities basing themselves on the frequentist and experiential categories which reach maximum values in this group. One is thus in the presence of an incipient experiential indeterminism.

Uncertainty

This group is the one that achieved the best recognition of randomness. Its members base this recognition on the unpredictability or uncertainty of the event, which category clearly surpasses its overall mean value. This therefore was the reason for the choice of the name of the group. They also base their recognition on multiplicity, whose value also is greater than the mean. This does not mean, however, that causal arguments are discarded, since the causality category surpassed the overall mean, while the subjective category reached its minimum in this group. With regard to the incorrect rejection of randomness, the values taken by the various categories are minimal. The estimation of probability is based on the equiprobability category, which reaches its maximum value in this group, well above the overall mean, and in this facet one clearly observes a failure to discriminate situations in which the principle of indifference is or is not applicable. The students consider the result of the phenomenon to depend on chance, and therefore that the outcomes are equally probable. Thus, one can conclude that this group represents a naïve indeterminism because they recognize the uncertain nature of the situations they are presented with, but fail to adequately address them.

Contingency

This group is second in relation to the recognition of randomness. There stands out the presence of arguments based on multiplicity, which reaches the maximum value in this group. The exact same thing is the case with causality. However, the commonest category is uncertainty, with a higher value than its overall mean. In the incorrect rejection of randomness, the multiplicity and uncertainty categories reached their minimum values in this group. When they have to estimate probability, they argue from contingency and Laplacian standpoints, categories which reach their maximum values in this group. Also, they use the frequentist category with a value slightly above the overall mean, whereas the experiential and equiprobability categories are those with their minimum values in this group. This trend of thought is that with the highest level of complexity. It recognizes the random nature of the situations that were presented in the questionnaire with arguments corresponding to different categories, and estimates probabilities with arguments that refer to an objective conception of probability.

CONCLUSION

The results confirm a certain distance between the prospective mathematics teachers' mental models and the standard conceptual models established in probability theory. This suggests that the teaching of this theory is deficient in that it does not give the students the opportunity to encounter different points of view about the theory's nature and uses. There is therefore a need for changes to be made to the curriculum and to the instruction the students are given in order to contribute to treating chance in depth, with probability theory being approached from its varied perspectives in terms of up-to-date educational and conceptual theoretical frameworks.

We believe that the results of this research constitute a further contribution to Professional Knowledge, and will therefore be of use to those responsible for the education of future mathematics teachers given the importance of this subject in the Probability and Statistics curriculum (Serradó, Azcárate & Cardeñoso, 2006).

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